METRIC GRAPH THEORY

JAIST (Japan Advanced Institute of Science and Technology) Workshop on Metric Graph Theory will take place in Kanazawa, Japan, during November 11-13,

Organizers:

Michel Deza (ENS, Paris, and JAIST) and Tatsuro Ito (Kanazawa University) with support of the 7th Japan Conference on Computational Geometry and Graphs http:www.jaist.ac.jp/~uehara/JCCGG09/cfp.html

Confirmed Invited Speakers:

Eiichi Bannai (Kyushu University)

Alexander Ivanov (Imperial College, London)

Sandi Klavzar (University of Ljubljana)

Jack Koolen (Pohang University of Science and Technology)

Sergey Shpectorov (University of Birmingham)

Egon Schulte (Northeastern University, Boston)

Yaokun Wu (Shanghai Jiao Tong University)

Paul-Hermann Zieschang (University of Texas at Brownsville)

Conference Site: Kanazawa Culture Hall, 15-1 Takaoka-cho, Kanazawa, 5 min. walk from central hotel "Toyoko Inn", where participants stay.

The lectures, each 50 minutes, will be in 3rd Conference Room.

SCHEDULE

11 November

10:00-10:50 Eiichi Bannai
11:00-11:50 Egon Shulte
12:00-13:15 LUNCH
13:30-14:20 Jack Koolen
14:30-15:20 Yaokun Wu

12 November

10:00-10:50 Sandi Klavzar
11:00-11:50 Sergey Shpectorov
12:00-13:15 LUNCH
13:30-14:20 Paul-Hermann Zieschang
14:30-15:20 Alexander Ivanov
15:30-16:20 Tatsuro Ito

ABSTRACTS

Title: Examples and classifications of certain Euclidean t-designs
Speaker: **Eiichi Bannai, Kyushu University**

Abstract:
Euclidean t-designs were defined by Neumaier-Seidel (1989) as a two step generalization of spherical designs. A natural lower bound for the cardinality of a Euclidean t-design is known, and tight t-design is defined as a t-design whose cardinality is equal to this lower bound. In this talk, the following two topics (1) and (2) will be discussed.

(1) We show that a coherent configuration is attached to a tight Euclidean t-design (for arbitrary t) on two concentric spheres. Then we study the classification problem of certain tight or close to tight t-designs on two concentric spheres (for some small t). (2) Through our attempt (yet to be completed) to classify Euclidean tight 6-designs on two concentric spheres with one layer (fiber) being a spherical tight 4-design, we have found a new example of such Euclidean tight 6-design in $\mathbb{R}^{22}$. We describe this new example, and we also prove the uniqueness of it. (The above (1) is based on joint work with Etsuko Bannai, and (2) is based on joint work with Etsuko Bannai and Junichi Shigezumi.)

Title: Towards the classification of P- and Q- polynomial association schemes

Speaker: **Tatsuro Ito, Kanazawa University**

Abstract:

Based on the recent development of the representation theory of Terwilliger algebras, I will discuss how I plan to attack the classification problem of P- and Q-polynomial association schemes.

Title: Majorana Representations of $L_3(2)$
Speaker: **Alexander Ivanov, Imperial College, London**

Abstract:

The Monster group M acts on a real vector space V of dimension 196,884 which is the sum of a trivial 1-dimensional module and a minimal faithful module. There are M-invariant scalar product $(\ ,\ )$ on V, an M-invariant bilinear commutative non-associative algebra product $\cdot$ on V (commonly known as the Conway–Griess–Norton algebra), and a subset A of $V^{\#}$ indexed by the conjugacy class of $2A$-involutions in M. Certain properties of the tuple

$$(M, V, A, (\ ,\ ), \cdot)$$

were axiomatized under the name of *Majorana representation* in Chapter 8 of A.A.Ivanov, *The Monster Group and Majorana Involutions*, Cambridge Univ. Press, Cambridge 2009.

This enables one to speak about such a representation for an arbitrary group G generated by involutions. A representation might or might not exist, but it does exist whenever G is a subgroup in M generated by its $2A$-involutions. We say that the latter representation is *based on embedding* of G into the Monster. The essential motivation for introducing the Majorana terminology was the most remarkable result by S. Sakuma published in 2007, which provides a classification of the Majorana representations of the dihedral groups.

There are nine such representations and every single one is based on an embedding into the Monster of the relevant dihedral group. It is a well known fundamental property of the Monster that its 2A-involutions form a class of 6-transpositions and that there are precisely nine M-orbits on the pairs of $2A$-involutions (hence also on the $2A$-generated dihedral subgroups in M). I am intended to report on recent classification of the Majorana representations of $L_3(2)$. There are two such represntations of dimensions 21 and 49, each of them is based on an embedding of $L_3(2)$ into the Monster.

Title: Metric and complexity properties of the Fibonacci dimension of a graph

Speaker: **Sandi Klavžar, University of Ljubljana, University of Maribor and UMFM, Slovenia**

Abstract:

A *Fibonacci string* is a binary string without two consecutive ones. The *Fibonacci cube* Γ_d, $d \geq 1$, is the subgraph of the d-cube Q_d induced by the Fibonacci strings of length d. The *Fibonacci dimension* $\mathrm{fdim}(G)$ of a graph G is the smallest integer f such that G admits an isometric embedding into the f-dimensional Fibonacci cube.

Graphs with finite Fibonacci dimension (as well as with finite lattice dimension) are precisely partial cubes. Bounds on the Fibonacci dimension of a graph G in terms of its isometric dimension $\mathrm{idim}(G)$ and the lattice dimension $\mathrm{ldim}(G)$ will be given. For instance, $\mathrm{fdim}(G) \leq \mathrm{idim}(G) + \mathrm{ldim}(G) - 1$. The Fibonacci dimension will be combinatorially characterized using properties of an associated graph and the dimension will be determined for certain families of graphs.

From the complexity point of view it is a bit surprisingly that it is polynomial to decide whether $\mathrm{fdim}(G) = 2\,\mathrm{idim}(G) - 1$ but NP-complete whether $\mathrm{fdim}(G) = \mathrm{idim}(G)$. Moreover, it is also NP-hard to approximate $\mathrm{fdim}(G)$ within $(741/740) - \varepsilon$. On the other hand, there exists an $(3/2)$-approximation algorithm for $\mathrm{fdim}(G)$ in the general case and a $(1 + \varepsilon)$-approximation algorithm for simplex graphs.

This is a joint work with *Sergio Cabello* (University of Ljubljana) and *David Eppstein* (University of California).

Title: Optimal Realizations

Speaker: **Jack Koolen, Pohang University of Science and Technology**

Abstract:

Let (X, D) be a finite metric space. A realization of (X, D) is a weighted graph $G = (V, E, w : E \to \mathbf{R})$ such that $X \subseteq V$ and $d_G(x, y) = D(x, y)$ for all $x, y \in X$ where $d_G(x, y)$ is the shortest path distance in the graph. Note that $H = (X, \binom{X}{2}, w : xy \mapsto D(x, y))$ is always a realization of (X, D). It is called *optimal* if $\sum_{xy \in E} w(xy)$ is minimal among all realizations.

In this talk I will discuss recent developments on optimal realizations. (This talk is based on joint work with A. Lesser, V. Moulton, A. Dress, K.Huber and A. Spillner)

Title: The Skeletal Approach to Polyhedra and Symmetry
Speaker: **Egon Schulte, Northeastern University, Boston**

Abstract:
Symmetric polyhedra have been investigated since antiquity. With the passage of time, the concept of a polyhedron has undergone a number of changes which have brought to light new classes of highly-symmetric polyhedra. Coxeter's famous "Regular Polytopes" and his various other writings treat the Platonic solids, the Kepler-Poinsot polyhedra and the Petrie-Coxeter polyhedra in great detail, and cover what might be called the classical theory.

A lot has happened in this area in the past 35 years. Around 1975, Grunbaum initiated the skeletal approach to polyhedra and symmetry, which is essentially graph-theoretical. A polyhedron is viewed as a geometric (edge) graph in space equipped with additional structure imposed by the faces, and its symmetry is measured by transitivity properties of its geometric symmetry group.

This talk describes the complete enumeration of regular or chiral polyhedra in Euclidean 3-space. Regular, or reflexibly regular, polyhedra have a geometric symmetry group which is transitive on the flags. Chiral, or irreflexibly regular, polyhedra are nearly regular polyhedra; their geometric symmetry groups have two orbits on the flags such that adjacent flags are in distinct orbits. The geometry and combinatorics of these polyhedra is generally quite complicated. There are only 48 regular polyhedra but several infinite families of chiral polyhedra.

Speaker: **Sergey Shpectorov, University of Birmingham**
Title: On l1-embeddable quadrilateral and hexagonal tilings of the Mobius strip

Abstract:
In a recent paper, G. Wang and H. Zhang show that within a particular family of Möbius graphs only three small graphs are ℓ_1-embeddable. The graphs they consider are the 1-skeletons of the natural 2-parameter families of quadrilateral and hexagonal tilings of the Möbius strip. This result continues the study of surface graphs that are ℓ_1-embeddable or have similar metric properties.

In the current project, joint with G. Wang, we generalize the result of Wang and Zhang to arbitrary quadrilateral and hexagonal tilings of the Möbius strip. It turned out that for the hexagonal tilings the answer is essentially the same even in the general case. Namely, all hexagonal tilings with ℓ_1-embeddable 1-skeleton are small. In contrast, there are infinitely many quadrilateral tilings with ℓ_1-embeddable 1-skeleton. Nevertheless, we were able to show that all of them have a relatively simple structure, built around a single isometric cycle that is not nulhomotopic. Our description amounts to a complete classification of such tilings.

Title: Chordality and hyperbolicity of a graph
Speaker: **Yaokun Wu, Shanghai Jiao Tong University**

Abstract:
Let G be a connected graph with the usual shortest-path metric d. The graph G is δ-hyperbolic provided for any vertices x, y, u, v in it, the two larger of the three sums $d(u, v) + d(x, y), d(u, x) + d(v, y)$ and $d(u, y) + d(v, x)$ differ by at most 2δ. The graph G is k-chordal provided it has no induced cycle of length greater than k. Brinkmann, Koolen and Moulton find that every 3-chordal graph is 1-hyperbolic and is not $\frac{1}{2}$-hyperbolic if and only if it contains one of two special graphs as an isometric subgraph. For every $k \geq 4$, we show that a k-chordal graph must be $\frac{\lfloor \frac{k}{2} \rfloor}{2}$-hyperbolic and there does exist a k-chordal graph which is not $\frac{\lfloor \frac{k-2}{2} \rfloor}{2}$-hyperbolic. Moreover, we prove that a 5-chordal graph is $\frac{1}{2}$-hyperbolic if and only if it does not contain any of a list of six special graphs as an isometric subgraph.
This is a joint work with *Chengpeng Zhang* (Shanghai Jiao Tong University).

Title: Metrics on association schemes which lead to twin buildings

Speaker: **Paul-Hermann Zieschang, University of Texas at Brownsville**

Abstract:

Let S be an association scheme, let L be a set of involutions of S, and assume that $\langle L \rangle = S$. The scheme S is called a Coxeter scheme with respect to L if it is constrained with respect to L and if the metric induced by L satisfies the exchange condition (a word by word translation of the group theoretic exchange condition to scheme theory).

It has been shown that Coxeter schemes can be identified with buildings. Moreover, from [2; Theorem 12.3.4] one knows that finite Coxeter schemes arise from groups in a well-understood way if they do not contain nontrivial thin elements and if the underlying set of involutions has at least three elements.

In my talk, I will discuss the question whether there exists a class of schemes which can be identified with twin buildings in the sense of [1] in a similar way as Coxeter schemes can be identified with buildings.

[1] Tits, J.: *Twin buildings and groups of Kac-Moody type*, London Math. Soc. Lecture Note Ser. **165**, Cambridge University Press (1992)

[2] Zieschang, P.-H.: *Theory of Association Schemes*. Springer Monographs in Mathematics, Berlin Heidelberg New York (2005)